Airline Fleet Assignment and Schedule Planning

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ABSTRACT

Airline fleet assignment plays important role for efficient operation of airlines. In order to save costs from the procurement and maintenance of extra aircrafts, the airline operators would always seek to minimise the fleet size, while at the same time being able to operate all the flights without shortage of aircraft. Therefore, the daily flight schedule of the airline must be prepared in such a way that the destinations can be served with minimum number of aircrafts. Considering this, there is huge prospect of using optimisation techniques to select the optimum mix of various types of aircrafts in the airline fleet and thereby to produce an optimum flight schedule.

Keywords: Integer Linear Programming, Fleet Assignment Model (FAM) algorithms. Integrated Schedules Design (ISD), Extended Schedules Design (ESD) algorithms.

Introduction

In scheduled passenger air transportation, airline profitability is critically influenced by the airline's ability to construct flight schedules containing flights at desirable times in profitable markets. The schedule design problem involves selecting an optimal set of flight legs to be included in the schedule, while the fleet assignment problem involves assigning aircraft types (or fleets) to flight legs to maximize revenues and minimize operating cost simultaneously (Lohatepanont, 2001).

There is huge prospect of using optimisation techniques to select the optimum mix of various types of aircrafts in the airline fleet and thereby to produce an optimum flight schedule. In this paper, we will discuss and review an optimisation model that has been developed for the fleet assignment and flight scheduling that is applicable for existing airlines with a given fleet size of different aircraft categories. All the aforementioned models are capable of producing an optimum solution regarding the minimisation of the fleet size to operate in all the routes. A note for future research direction will also address in this paper.

Overview of Airline Fleet Planning

Airline planning process is decomposed into several sequential steps (Lohatepanont, 2001). As can be seen in Figure 1, the vertical bar describes the time horizon of this sequential process from several years out to a few days before flight departures. The right axis categorizes the nature of the decisions involved in this planning process, ranging from strategic at the top down to tactical decisions at the bottom. Note, however only fleet planning sequential approaches will be described in this paper.

Fleet Planning

Fleet planning is one of the most important strategic decisions and involves huge capital investment. There are two major approaches to fleet planning (Belobaba, 1999):

1. Top-Down Approach
2. Bottom Up Approach

The 'top-down' approach involves high level, system wide financial analysis of the impacts of options. This approach is most common in practice because it does not involve sophisticated models or detailed analysis.

The 'bottom-up' approach, however, required a series of detailed simulations of airline operations, ranging from route structure to operation. This approach depends heavily on the quality of the data, especially the detailed forecasts of future scenarios.

Schedule Planning

The schedule planning step typically begins 12 months before the schedule goes into operation and lasts approximately 9 months. In the beginning, the schedule planning step begins with route development, in which the airline decides which markets, defined by origins and destinations, it wants to serve, based primarily on wide demand system information. Most of the time, the schedule planning step starts from an existing schedule to reflect changing demands and environment, this is referred to as schedule development. The major components in the schedule development step are:

1. Schedule Design,
2. Fleet Assignment, and
3. Aircraft Rotations.

Schedule Design

The schedule design step is the most complicated step of all and traditionally has been decomposed into two sequential steps:

1. Frequency Planning, and
2. Timetable Development.

In frequency planning, planners determine the appropriate service frequency in a market. In timetable development, planners place the proposed services throughout the day subject to approximate network considerations and other constraints.

Fleet Assignment

The purpose of fleet assignment is to assign the available aircraft to every flight leg such that the seating capacity on the aircraft closely matches the demand for every flight. The assignment of aircraft to flight legs has to respect to conservation of aircraft flow, that is, an aircraft entering a station has to leave that station at some later point in time. If the schedule cannot be fleeted with the available number of aircraft, minor changes must be made to the schedule.

Aircraft Rotations

The purpose of aircraft rotation is to find a maintenance feasible rotation (or routing) of aircraft, given a fleeted schedule and the available number of aircraft of each type. A rotation is a sequence of connected flight legs that are assigned to a specific aircraft, beginning and ending at the same location, over a specific period of time. A maintenance feasible rotation is a routing of an aircraft that respects the maintenance rules of the airlines and regulatory agencies (Barnhart et.al, 2001).

Literature Review

There has been a lot of research on airline fleet assignment. The fleet assignment was proposed in early 1954 by Dantzig and Ferguson (1954) by using linear programming to fleet assignment problems considering for non-stop routes. They formulate the problem as a linear program thus allowing fractional solutions. However, fractional solutions might not be critical if the assignment is considered over some period of time.

Over the past few decades, this topic was extensively researched. Recent developments include Abara (1989) developed and solved the fleet assignment problem as an integer linear programming problem, permitting assignment of two or more fleets to a flight schedule simultaneously. Subramanian et al (1994) developed a fleet assignment model, which assigns fleet types (not individual aircraft tail numbers) to the flight legs for a hub and spoke type operation of the airline. Some researchers (Hane et al, 1995; Rushmeier and Kontogiorgis, 1997) modelled the fleet assignment as mixed integer multi commodity flow problem with side constraints defined on a time expanded network, which resulted in a faster solution. Barnhart et al (2002) proposed a new formulation to the fleet assignment problem and solution approach that captures network effects and generates superior solutions.

The basis for several fleet assignment models currently used by the airlines industry is the model proposed by Hane, et al (1995). They model the fleet assignment problem as a multicommodity network flow problem, where fleet types are to be assigned to flight legs in the network once, using only the available number of aircraft. Several problem size reduction techniques are devised, for example node consolidation and island construction. Node consolidation is used to reduce the number of nodes by separating a consolidated series of arrival nodes from a consolidated series of departure nodes. Island construction is employed mostly at spoke stations where flight connections occur sparsely during the day.

Airline Fleet Assignment Models

In this section, we will review Fleet Assignment Models (FAM) which commonly used by the airlines industry, namely Basic Fleet Assignment Model. The basic FAM serves as the basis for most of other variations. Before describing the model in detail, the complete list of notation as follow.

Notation

Sets

A : the set of airports indexed by *o*.

L : the set of flight legs in the flight schedule indexed by *i*.

K: the set of different fleet types indexed by *k*.

T : the sorted set of all event (departure or availability) times at all airports, indexed by t_j.

The event at time t_j occurs before the event at time t_{j+1} $|T|$=m

N: the set of nodes in the timeline network indexed by $\{k,o,t_j\}$

$N_{k,i}$: the set of copies of flight leg $i \in L$ for fleet type $k \in K$

CL(k): the set of flight legs that pass the count time when flown by fleet type *k*.

I(k,o,t): the set of inbound flight legs to node $\{k,o,t_j\}$.

O(k,o,t): the set of outbound flight legs from node $\{k,o,t_j\}$.

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Decision Variables

$$f_{k,i} = \begin{cases} 1, & \textit{if flight leg } i \in N \text{ is assigned to fleet type } k \in K \\ 0, & \text{otherwise} \end{cases}$$

$$f_{n,k,i} = \begin{cases} 1, & \textit{if} \text{ copy } n \in N_{k,i} \text{ of flight leg } i \in N \text{ is assigned to fleet type } k \in K \\ 0, & \text{otherwise} \end{cases}$$

$\boldsymbol{y_{k,o,t_j^+}}$ **:** the number of fleet type $k \in K$ aircraft that are on the ground at airport $o \in A$ immediately after time $t_j \in T$

$\boldsymbol{y_{k,o,t_j^+}}$ **:** the number of fleet type k aircraft that are on the ground at airport $o \in A$ immediately before time $t_j \in T$. If t_1 and t_2 are the times associated with adjacent events, then $\boldsymbol{y_{k,o,t_1^+}} = \boldsymbol{y_{k,o,t_2^-}}$

Parameters/Data

N_k : the number of aircraft in fleet type k, $\forall k \in K$

$C_{k,i}$: the assignment cost when fleet type , $k \in K$ is assigned to flight leg $i \in L$

$C_{n,k,i}$: the assignment cost when fleet type $k \in K$ is assigned to copy $n \in N_{ki}$ of flight leg $i \in L$

Data Input

Most fleet assignment models require three types of data input:

1. flight schedule
2. demand and fare data associated with the given flight schedule, and
3. fleet characteristics

Basic Fleet Assignment Model

The kernel of most Fleet Assignment Models can be described as:

maximize	:	fleeting contribution
(or minimize	:	assignment cost)
subject to	:	all flights flown by exactly one aircraft type

Or mathematically as:

$$Min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1, \forall i \in L$$

$$\boldsymbol{y_{k,o,t^-}} + \sum_{i \in I(k,o,t)} f_{k,i} - \boldsymbol{y_{k,o,t^+}} - \sum_{i \in O(k,o,t)} f_{k,i} = 0, \forall k, o, t$$

$$\sum_{o \in A} y_{k,o,t_m} + \sum_{i \in CL(k)} f_{k,i} \le N_k, \forall k \in K$$

$$f_{k,i} \in \{0,1\}, \forall k \in K, \forall i \in L$$

$$y_{k,o,t} \ge 0, \forall k, o, t$$

Constraint (2) *are cover constraints* ensuring that each flight is covered once and only once by a fleet type. Constraint (3) are *conservation of flow constraints* ensuring aircraft balance, that is, aircraft going into a station at a particular time must leave that station at some later time. Constraint (4) *are count constraints* ensuring that only the available number of aircraft of each type are used in the assignment. The objective function coefficient $C_{k,i}$ is the summation of the following components:

1. Operating costs
2. Carrying cost
3. Spill cost
4. Recaptured revenue

Note that FAM assumes flight leg independence. Specifically, the objective function coefficient, $C_{k,i}$, is determined for assignment of fleet type k to flight leg i independently of any other flight legs in the network.

Variations on fleet assignment approaches can be found in Dantzig (1954), Daskin and Panayotopoulos (1989), Abara (1989), Berge and Hopperstad (1993), Clarke et.al (1996), Talluri (1996), Rushmeier and Kontogiorgis (1997), Barnhart et. al (1998) and Lohatepanont (2001), for example.

FAM Solution

Hane et. al (1995) demonstrate solution techniques for this model using an airline network with 2600 flights and 11 fleet types. The techniques they employ include:

1. *node consolidation*: an algebraic substitution technique that results in significant reductions in problem size;
2. *island construction*: an exploitation of special problem structure that achieves further reduction in problem size; and
3. *specialised branching strategies and priorities*: branching based on special ordered sets (SOS) and selection of variables on which to branch based on a measure of variability of the objective coefficients.

In summary, there are several aspects of the problem that are modeled only approximately or entirely ignored, hence, room for improvement exists. The basic fleet assignment model by Hane, et al (1995), in particular, will serve as a basis for development and discussion throughout this paper.

Scheduled Design and Fleet Assignment

Generating an optimal schedule for any given period is of utmost interest and importance to the airlines. In the past, these tasks have hadbeen separated and optimized in a sequential manner, because the integrated model to optimize the entire process is wasunsolvable. Today, advanced technologies and better understanding of the problems have allowed operations researchers to begin integrating and globally optimizing these sequential tasks.

Demand and Supply Interactions

Demand and supply interaction is a crucial element in the construction of an airline schedule. Understanding this element is essential for the development of an efficient flight schedule.

Demand

The demand for air travel is a derived demand (Simpson and belobaba, 1992); it is derived from other needs of individuals. For the purpose of schedule design and fleet assignment, a market is defined by an origin and destination pair. There are alternative ways to estimate total market demand for air travel. Teodovoric (1989) details a methodology for estimating total air travel demand using a classical four step transport planning process, namely:

1. Trip generation;
2. Trip distribution;
3. Modal split; and
4. Trip assignment.

For the purpose of schedule design for a given airline, we are interested in the unconstrained market demand, that is, the maximum fraction of the total demand in a market, termed market share, that the airline is able to capture.

Supply

To compete for market share, the airline develops its flight network. The first step in developing the flight network is to adopt an appropriate network structure. Simpson and Belobaba (1992) present three basic network structures, namely:

1. a linear network;
2. a hub and spoke network;
3. a point to point (complete) network.

Figure 4 depicts these network structures for 4 locations (nodes).

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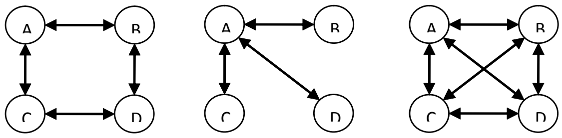

(a) Linear Network (b) Hub and Spoke Network (c) Complete Network

Figure 4. Basic Network Structures

Hub and spoke network has been adopted by most major U.S airlines since their deregulation in 1978 (Wheeler, 1989). Its main advantage derives from connecting opportunities at the hub airport enabling airlines to consolidate demand from several markets onto each flight. This enables airlines to serve more markets especially when the demans in some markets do not warrant direct services. Simpson and Belobaba (1992) note that the hub and spoke network structure creates more stable demand at the flight leg level. By mixing and consolidating demands from deifferent markets on each flight leg, the hub and spoke network can reduce variations in the number of passengers at the flight leg level, because market have demand distributions.

Integrated Models for Schedule Design and Fleet Assignment

Barnhart et. al (2001) proposed the development of integrated models for airline schedule design and fleet assignment for two markets (constant markets and variable markets). They assume their schedule is daily, that is, the schedule repeats everyday. Because conservation of aircraft is always maintained, they can count the number of aircraft in the network, by taking a snapshot of the network at a pre-specified point in time and counting the number of aircraft both in the air and on the ground at stations.

Previous works on integrated schedule design and fleet assignment approaches can be found in Chan (1972), Simpson (1966), Soumis, Ferland, and Rousseau (1980), Dobson and Lederer (1993), Marsten et.al (1996), and Berge (1994), for example.

Notation

Sets

P : the set of itineraries in a market indexed by p or r.

P^0 : the set of optional itineraries indexed by q.

A : the set of airports indexed by o.

L : the set of flight legs in the flight schedule indexed by i.

L^F : the set of mandatory flight indexed by i.

L^O : the set of optional flight indexed by i.

K: the set of different fleet types indexed by k.

T : the sorted set of all event (departure or availability) times at all airports, indexed by t_j.

The event at time t_j occurs before the event at time t_{j+1} $|T|$=m

N: the set of nodes in the timeline network indexed by $\{k,o,t_j\}$

$CL(k)$: the set of flight legs that pass the count time when flown by fleet type k.

$I(k,o,t)$: the set of inbound flight legs to node $\{k,o,t_j\}$.

$O(k,o,t)$: the set of outbound flight legs from node $\{k,o,t_j\}$.

$L(q)$: the set of flightlegs in itinerary q.

Decision Variables

t_p^r: the number of passengers requesting itinerary p but the airline attempts to redirect to itinerary r.

$$f_{k,i} = \begin{cases} 1, & if\ flight\ leg\ i \in N \text{ is} \\ & \text{assigned to fleet type } k \in K \\ 0, & \text{otherwise} \end{cases}$$

$$Z_q = \begin{cases} 1, & if \text{ itinerary } q \in P^o \text{is selected;} \\ 0, & \text{otherwise} \end{cases}$$

$\boldsymbol{y_{k,o,t_j^+}}$ **:** the number of fleet type $k \in K$ aircraft that are on the ground at airport $o \in A$ immediately after time $t_j \in T$

$\boldsymbol{y_{k,o,t_j^+}}$ **:** the number of fleet type k aircraft that are on the ground at airport $o \in A$ immediately before time $t_j \in T$. If t_1 and t_2 are the times associated with adjacent events, then $\boldsymbol{y_{k,o,t_1^+} = y_{k,o,t_2^-}}$

Parameters/Data

CAP_i : the number of seats available in flight leg i (assuming fleeted schedule)

$SEATS_k$: the number of seats available in aircraft of fleet type k

N_k : the number of aircraft in fleet type k, $\forall k \in K$

N_q : the number of flight legs in itinerary q.

D_p : the unconstrained demand for itinerary p, i.e., the number of passengers requesting itinerary p.

Q_i : the unconstrained demand on leg i when all itineraries are flown.

$fare_p$: the fare for itinerary p

$\widetilde{fare_p}$: the carrying cost adjusted fare for itinerary p.

b_p^r: recapture rate from p to r; the fraction of passengers spilled from itinerary p that the airline succeeds in redirecting to itinerary r.

$\delta_i^p = \begin{cases} 1, & if \text{ itinerary } p \in P \text{ includes flight leg } i \in N; \\ 0, & \text{otherwise} \end{cases}$

ΔD_q^p : demand correction term for itinerary p as a result of cancelling itinerary q

Schedule Design with Constant Market Share

Integrated Schedule Design and Fleet Assignment (ISD-FAM) is commonly term for constant market share model. In this model, the assumptions made is the market shares of the carrier are constant, that is, although changes are made to the schedule, the unconstrained market demands of the carries of interest are not affected.

ISD-FAM is built upon the Itinerary based Fleet Assignment Model (IFAM) by Barnhart, Kniker, and Lohatepanont (2001). The model assume that markets are independent of one another, that is, demands in any market do not interact with demands in any other markets. This enables them to adjust demand for each market only if the schedule for that market is altered.

Objective Function

The objective of ISD-FAM is to maximize schedule contribution, defined as revenue generated less operating cost incurres. The operating cost of a schedule, denoted **O**, can be computed as $\sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i}$ once fleet-flight assignments are determined. The total revenue of a schedule can be computed from the folowing components:

1. Initial unconstrained revenue (**R**)

$$\mathbf{R} = \sum_{p \in P} fare_p D_p,$$

2. Lost revenue due to spill (**S**)

$$\mathbf{S} = \sum_{p \in P} \sum_{r \in P} fare_p t_p^r, \text{ and}$$

3. Recaptured revenue from recapturing spilled passengers (**M**)

$$\mathbf{M} = \sum_{p \in P} \sum_{r \in P} \widetilde{b_p^r fare_r} t_p^r$$

Equation (1) computes the initial unconstrained revenue for the schedule given unconstrained demand associated with all optional flight legs flown. Equations (2) and (3) measure the changes in revenue due to spill and recapture, respectively.

Formulation :

$$Min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (\widetilde{fare}_p - \widetilde{b_p^r} \widetilde{fare}_r) t_p^r$$

Subject to :

$$\sum_{k \in K} f_{k,i} = 1, \forall i \in L^F$$

$$\sum_{k \in K} f_{k,i} \leq 1, \forall i \in L^O$$

$$\boldsymbol{y_{k,o,t^-}} + \sum_{i \in I(k,o,t)} f_{k,i} - \boldsymbol{y_{k,o,t^+}} - \sum_{i \in O(k,o,t)} f_{k,i} = 0, \forall \{k,o,t\} \in N$$

$$\sum_{o \in A} y_{k,o,t_m} + \sum_{i \in CL(k)} f_{k,i} \leq N_k, \forall k \in K$$

$$\sum_{k \in K} CAP^k f_{k,i} + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p \widetilde{b_r^p} t_r^p \geq Q_i, \forall i \in L$$

$$\sum_{r \in P} t_p^r \leq D_p, \forall p \in P$$

$$f_{k,i} \in \{0,1\}, \forall k \in K, \forall i \in L$$

$$y_{k,o,t} \geq 0, \forall \{k,o,t\} \in N$$

$$t_p^r \geq 0, \forall p, r \in P$$

Constraints (2) are cover constraints for mandatory flights ensuring that every manadatory flight is assigned to a fleet type. Constraints (3) are conver constraints for optional flights allowing the model to choose whether or not to fly flight i in the resulting schedule; if flight i is selected, a fleet type has to be assigned to it. Constraints (4) ensure the conservation of aircraft flow. Constraints (5) are count constraints ensuring that only available aircraft used. Constraints (6) are capacity constraints ensuring that the number of passengers on each flight i does not exceed its capacity. Constraints (7) are demand constraints ensuring that we do not spill more passengers demand for the itinerary.

Schedule Design with Variable Market Share

In this section we present the Extended Schedule Design and Fleet Assignment Model (ESD-FAM) proposed by Barnhart et.al (2001), in which market shares are simultatneously updated as changes are made to the schedule.

Objective Function

Like in ISD-FAM, all average unconstrained itinerary demands are computed for the schedule with all optional flights flown. The objective of ESD-FAM is to maximize schedule contribution, defined as revenue generated less operating cost incurred. As explained in the previous section, an additional terms is requireddue to the introduction of demand correction terms:

changes in unconstrained revenue due to market share changes because of flight
leg addition or deletion (ΔR),

$$\Delta\mathbf{R} = \sum_{q \in P^O} (fare_q D_q - \sum_{p \in P: p \neq q} fare_p \Delta_q^p) \cdot (1 - Z_q)$$

The term $fare_q D_q$ is the total unconstrained revenue of itinerary q. The term $\sum_{p \in P: p \neq q} fare_p \Delta_q^p$ is the total change in unconstrained revenue on all other itineraries p ($\neq q$) in the same market due to deletion of itinerary q. Recall that Z_q equals 1 if q is flown and 0 otherwise. Thus, equation is the change in unconstrained revenue due to the deletion of itinerary q.

Formulation

ESD-FAM can be formulated as shown below.

$$Min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (\widetilde{fare}_p - \widetilde{b_p^r} \widetilde{fare}_r) t_p^r + \sum_{q \in P^O} (fare_q D_q - \sum_{p \in P: p \neq q} fare_p \Delta_q^p) \cdot (1 - Z_q)$$

subject to :

$$\sum_{k \in K} f_{k,i} = 1, \forall i \in L^F$$

$$\sum_{k \in K} f_{k,i} \leq 1, \forall i \in L^O$$

$$\boldsymbol{y_{k,o,t^-}} + \sum_{i \in I(k,o,t)} f_{k,i} - \boldsymbol{y_{k,o,t^+}} - \sum_{i \in O(k,o,t)} f_{k,i} = 0, \forall \{k,o,t\} \in N$$

$$\sum_{o \in A} y_{k,o,t_m} + \sum_{i \in CL(k)} f_{k,i} \leq N_k, \forall k \in K$$

$$\sum_{p\in P}\sum_{q\in P^O}\delta_t^p \Delta D_q^p(1-Z_q)+\sum_{k\in K} CAP^k f_{k,i}+\sum_{r\in P}\sum_{p\in P}\delta_i^p t_p^r-\sum_{r\in P}\sum_{p\in P}\delta_i^p \widetilde{b_r^p} t_r^p \geq Q_i, \forall i \in L$$

$$\sum_{q\in P^O}\Delta D_q^p(1-Z_q)+\sum_{r\in P} t_p^r \leq D_p, \forall p \in P$$

$$Z_q-\sum_{k\in k} f_{k,i} \leq 0, \forall i \in L_{(q)}$$

$$Z_q-\sum_{i\in L_{(q)}}\sum_{k\in k} f_{k,i} \geq 1-N_q, \forall q \in P^O$$

$$f_{k,i} \in \{0,1\}, \forall k \in K, \forall i \in L$$

$$Z_q \in \{0,1\}, \forall q \in P^O$$

$$y_{k,o,t} \geq 0, \forall\{k,o,t\} \in N$$

$$t_p^r \geq 0, \forall p,r \in P$$

Constraints (2) to (5) are similar to ISD-FAM. The term $\sum_{q\in P^O}\Delta D_q^p(1-Z_q)$ in Constraints (7) corrects the unconstrained demand for itinerary $p \in P$ when optional itineraries$q \in P^O$ are deleted. Similarly the term $\sum_{p\in P}\sum_{q\in P^O}\delta_t^p \Delta D_q^p(1-Z_q)$ in Constraints (6) represents corrected demand but at the flight level. Constraints (8) – (9) are itinerary status constraints that control the {0,1} variable Z_q, for itinerary q.

Demand corrections can be innacurate when two or more itineraries are cancelled at the same time. These inaccuracies can be obviated by adding another set of {0,1} variables indicating the status combinations of itineraries and associating additional demand correction terms with these variables.

Summary

In this section we reviewed two integrated models for airline schedule design and fleet assignment:

1. the integrated schedule design and fleet assignment model (ISD-FAM),
2. the extended schedule design and fleet assignment model (ESD-FAM).

ESD-FAM utilizes demand correction terms to adjust carrier market shares as schedules are altered. ISD-FAM, on other hand, ignores these complicated interactions and instead utilizes recapture rates to adjust demand, assuming constant market share.

Note for Future Research Direction

In this paper, we present and review basic FAM and Integrated Scheduled Design and Fleet Assignment (IFAM) model. As with any modelling, a number of assumptions should be made in efforts to tackle the schedule design and fleet assignment problems. These assumptions are necessary to simplify the problem and increase tractability, while others are made to facilitate operation. Future research should see some relaxation of these assumptions. If full relaxation in infeasible, measures to validate included assumptions are needed.

Demand and supply interaction issues in the schedule design problem represent a major research area that is yet to be fully investigated, understood and modeled. With better understanding of these interactions, efficient modelling techniques can be developed to tackle schedule design problem. Competitor's reactions may also be modeled in the form of demand and supply interactions.

In addition, current ISD-FAM and ESD-FAM proposed by Barnhart et.al (2001) ignore a number of operational issues. These issues include, for example, maintenance of hub structure airline presence in markets, minimum or maximum frequency in markets, gate and slots availability, etc. Thus, an operational model requires addition of these considerations to ensure an appropriate schedule.

Last, all the aforementioned fleet assignment model works are applicable for existing airlines with a given fleet size of different aircraft categories. However, if a new airline is going to be established to serve some predetermined destinations with known demand and decided aircraft types in each one, the above models are not capable of producing an optimum solution regarding the minimisation of the fleet size

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to operate in all the routes. Therefore, it is necessary to develop a new optimisation model to solve this problem for future work.

Notes:

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Appendix

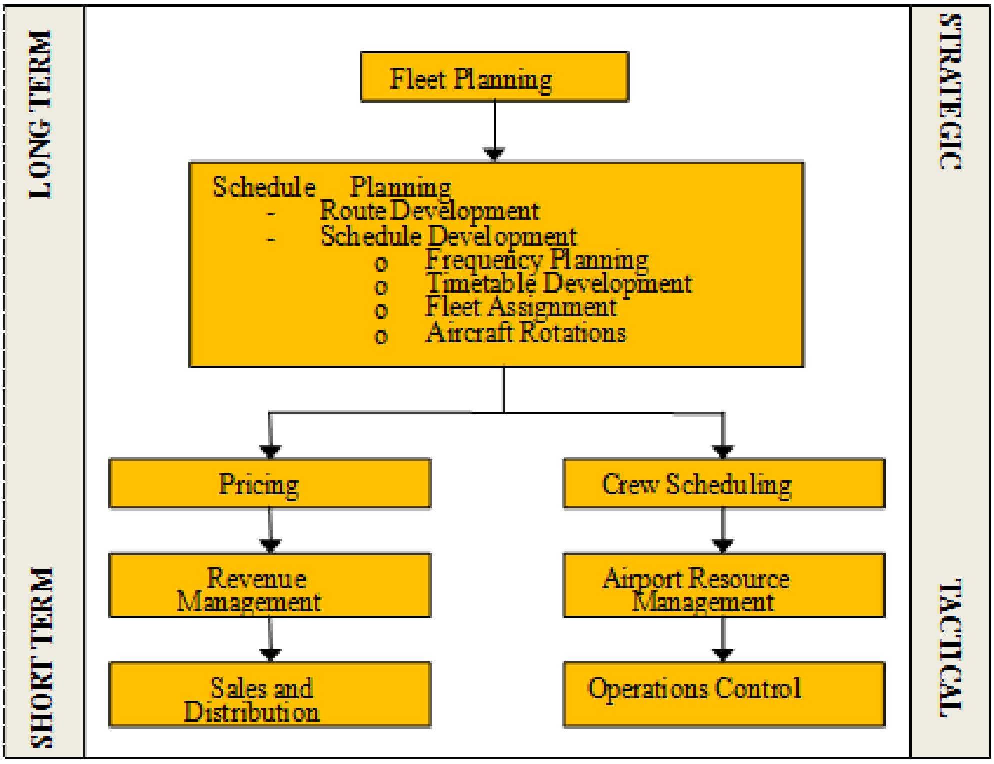

Figure 1 Airline Planning Process